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Effects of thermal conductivity in radiative magnetohydrodynamic channel flow

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SUMMARY

The Hartmann problem for the viscous laminar flow of an electrically conducting liquid between parallel walls with a transverse magnetic field is examined when effects of both thermal conductivity and radiation are significant. Details of the temperature distribution and radiative flux are presented.

1. Introduction

In a recent paper [1] the present author has studied the effects of thermal radiation upon Hartmann flow when thermal conductivity is insignificant. The importance of this case lies in the fact that then the governing equations may be solved exactly so that explicit forms for the temperature distribution and radiative flux may be obtained.

However such a model suffers from the serious defect that, in reality, it is most unlikely that thermal conductivity will be negligible if thermal radiation is not. Since in the absence of thermal conductivity no mechanism is present which can adjust the temperature of the fluid to that of an adjacent boundary, one finds the temperature profile discontinuous at its end points with a failure to match the temperature of the bounding wall. By analogy with the similar behaviour of the velocity in non-viscous flows it is usually considered that this unrealistic behaviour may be removed by the introduction of a thermal boundary layer.

In this paper the full system of governing equations for thermally conducting radiative Hartmann flow are treated without further approximation and numerical solutions obtained from which the precise nature of the effect of thermal conductivity may be deduced. The basic problem here studied is identical to that formulated in the earlier paper to which reference should be made for details. We proceed here merely to restate the fundamental equations governing the flow as presented there and then to develop the numerical solution.

2. The governing equations

A liquid of electrical conductivity σ, thermal conductivity k and coefficient of viscosity η is taken to flow between two infinite parallel insulating flat walls distance $2h$ apart with transverse applied magnetic field B_0 and orthogonal electric field E_0. If y denotes the distance from mid channel taken normally to the walls and all flow properties are assumed functions of y alone then with a pressure gradient p_0 down the channel one may derive the basic equations for the velocity, u, and temperature, T, as follows

$$\frac{d}{dy}\left(\eta\frac{du}{dy}\right) - \sigma B_0^2 u + (p_0 - \sigma E_0 B_0) = 0\,, \tag{1}$$

$$\frac{d}{dy}\left(k\frac{dT}{dy}\right) + \eta\left(\frac{du}{dy}\right)^2 - \frac{dq}{dy} + \sigma(E_0 + uB_0)^2 = 0\,. \tag{2}$$

For the present one-dimensional problem the differential approximation to the exact equation of radiative transfer for the radiative flux, q, may be employed, *viz.*

$$\frac{d^2 q}{dy^2} - 3\alpha^2 q = \frac{d}{dy}(\tilde{\sigma}\alpha T^4) \tag{3}$$

where α is a volumetric absorption coefficient and $\tilde{\sigma}$ is Stefan's constant.

The boundary conditions appropriate to these equations are able to be stated quite simply. If the lower wall at $y=-h$ has emissivity ε_1 and temperature T_1 whilst the upper wall at $y=h$ has emissivity ε_2 and temperature T_2 both walls being taken at rest these conditions are

$$u=0\,, \quad y=\pm h \tag{4}$$

$$T=T_1\,, \quad y=-h\,; \qquad T=T_2\,, \quad y=+h \tag{5}$$

$$\left(\frac{4}{\varepsilon_1}-2\right)q - \frac{1}{\alpha}\frac{dq}{dy} = 0\,, \qquad y=-h \tag{6}$$

$$\left(\frac{4}{\varepsilon_2}-2\right)q + \frac{1}{\alpha}\frac{dq}{dy} = 0\,, \qquad y=+h\,. \tag{7}$$

Clearly equation (1) with boundary conditions (4) separates from the remaining equations to yield the velocity, the distribution of which is unaffected by either thermal conductivity or radiation.

It is useful to introduce the following notation

$$P = \left(\frac{p_0 h\mu}{B_0^2}\right)\left(\frac{1}{\mu h\sigma U_0}\right) = \frac{\text{pressure ratio}}{\text{magnetic Reynolds number}}\,,$$

$$J = \left(\frac{j_0 h\mu}{B_0}\right)\left(\frac{1}{\mu h\sigma U_0}\right) = \frac{\text{current ratio}}{\text{magnetic Reynolds number}}\,,$$

$$M = B_0 h\left(\frac{\sigma}{\mu}\right)^{\frac{1}{2}} = \text{Hartmann number}\,,$$

which are related by

$$P = J + \frac{1}{M\coth M - 1}\,.$$

In addition we write

$$\Theta = \frac{T_2}{T_1} = \text{temperature ratio across the channel}\,,$$

$$\omega = \alpha h = \text{Bouguer number}\,,$$

$$N = \left(\frac{\tilde{\sigma}T_1^4}{\rho_0 U_0^3}\right)\left(\frac{h\rho_0 U_0}{\eta}\right) = \frac{\text{Reynolds number}}{\text{Boltzmann number}}\,,$$

$$\nu = \left(\frac{U_0^2}{c_p T_1}\right)\left(\frac{\eta c_p}{k}\right) = (\text{Eckert number})(\text{Prandtl number})\,,$$

$$\theta = \frac{T}{T_1} = \text{dimensionless temperature}\,,$$

$$Q = \frac{q}{\tilde{\sigma}T_1^4} = \frac{\text{radiative flux}}{\text{black wall emissive power}}\,,$$

$$\zeta = \frac{y}{h} = \text{dimensionless channel coordinate}\,.$$

Then with the insertion of the form for the velocity derived from equation (1) the remaining equations (2) and (3) which govern the problem may be written as the first order system

$$\frac{d\theta}{d\zeta} = \psi$$

$$\frac{dQ}{d\zeta} = \phi$$

$$\frac{d\psi}{d\zeta} = N\nu\phi - M^2\nu\left\{P^2 + \frac{M^2(P-J)^2\cosh 2M\zeta}{\sinh^2 M} - \frac{2MP(P-J)\cosh M\zeta}{\sinh M}\right\}$$

$$\frac{d\phi}{d\zeta} = 3\omega^2 Q + 16\omega\theta^3\psi$$

where the boundary conditions (5)–(7) become

$$\theta = 1\,, \quad \phi = \omega\left(\frac{4}{\varepsilon_1} - 2\right)Q\,, \quad \text{at } \zeta = -1\;;$$

$$\theta = \Theta\,, \quad \phi = -\omega\left(\frac{4}{\varepsilon_2} - 2\right)Q\,, \quad \text{at } \zeta = +1\,.$$

3. Analysis

The solution of the above equations under the specified boundary conditions forms a two point boundary value problem of considerable magnitude. It proves to be impossible to obtain a solution within a reasonable time using standard procedures of numerical analysis and a digital computer, for other than trivial values of the parameters. Accordingly recourse was had to analogue methods of computation which, despite extreme sensitivity of the solution at

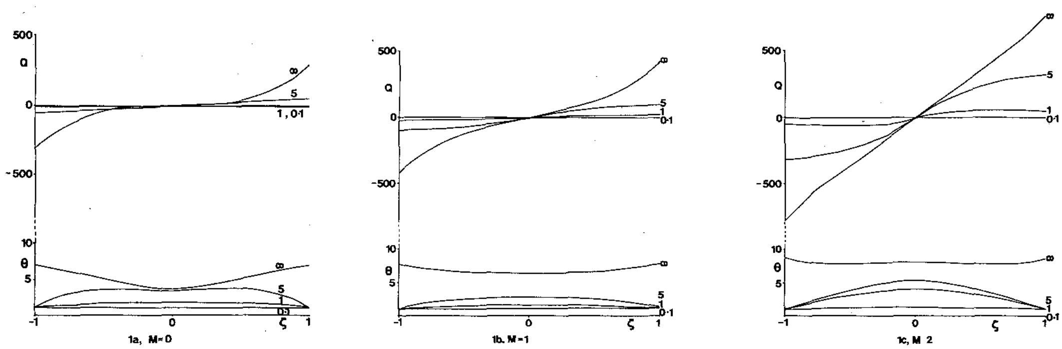

Figure 1. Radiative flux and temperature $\varepsilon_1 = 1$, $\varepsilon_2 = 1$, $\Theta = 1$. All cases $J = 1$, $\omega = 0.1$, $N = 0.01$. As labelled $\nu = 0.1$, 1, 5, ∞.

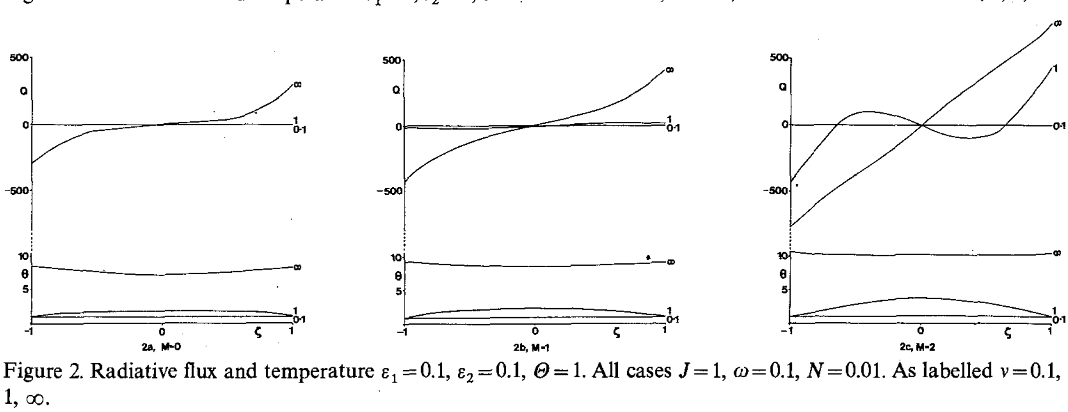

Figure 2. Radiative flux and temperature $\varepsilon_1 = 0.1$, $\varepsilon_2 = 0.1$, $\Theta = 1$. All cases $J = 1$, $\omega = 0.1$, $N = 0.01$. As labelled $\nu = 0.1$, 1, ∞.

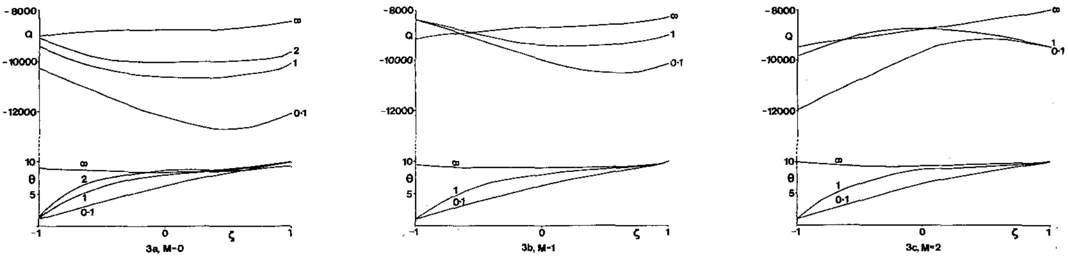

Figure 3. Radiative flux and temperature $\varepsilon_1=1$, $\varepsilon_2=1$, $\Theta=10$. All cases $J=1$, $\omega=0.1$, $N=0.01$. As labelled $\nu=0.1$, 1, 2, ∞.

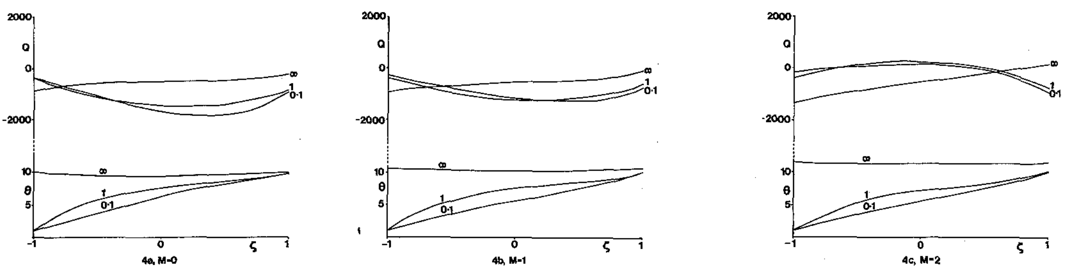

Figure 4. Radiative flux and temperature $\varepsilon_1=0.1$, $\varepsilon_2=0.1$, $\Theta=10$. All cases $J=1$, $\omega=0.1$, $N=0.01$. As labelled $\nu=0.1$, 1, ∞.

one end of the range to changes in the boundary values taken at the other end, and thus difficulty in fitting the overall boundary conditions, were successfully employed.

The distributions for radiative flux and temperature are presented in figures 1 to 4 in the case $J=1$ corresponding to an electromagnetic brake for three values of Hartmann number and a variety of wall emissivities ε_1, ε_2 and temperature ratio Θ with constant radiative parameters ω and N. In each instance values of the thermal conductivity parameter ν have been chosen representative of various degrees of thermal conductivity together with that, $\nu=\infty$, appropriate to the non-conducting case. The graphical representations are self-explanatory and in need of no further general comment, but it is worth noting the strong indications of the existence of a thermal boundary layer when the thermal conductivity is low, an effect which, as expected, is unaffected by the presence of radiation.

Finally it is a pleasure to acknowledge the assistance of Mr. T. Jenkins of the Computing Laboratory in the University of Bradford whose expert manipulation of the EAL 680 analogue computer in this university produced the above solutions.

REFERENCE

[1] J. B. Helliwell, Effects of thermal radiation in magnetohydrodynamic channel flow, *J. Eng. Math.*, 7 (1973) 11–17.